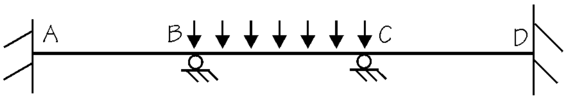
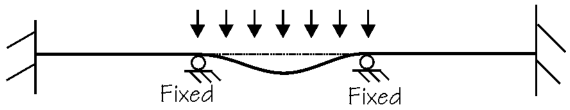
Moment-Distribution

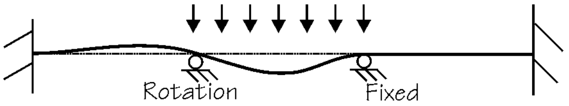
The method of moment distribution relies on a series of calculations that are repeated and that with every cycle come closer to the final situation. In this way we are able to avoid solving simultaneous equations. Inspection of the slope-deflection equations shows us that the final end-moments depend on 4 effects namely, θ_A, θ_B, ψ_{AB} and the fixed end moments, FEM. By using moment-distribution we are able to investigate each effect separately. The following beam will be used to illustrate moment-distribution.



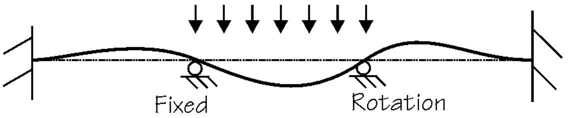
Rotation is possible at both B and C



Rotation at B and C are prevented and the load is applied. FEM will result. These are called the initial moments.



Allow B to rotate until moment equilibrium is reached. Rotation at B will induce a moment at C.

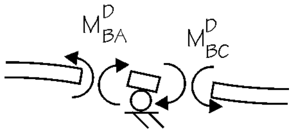


Allow C to rotate until moment equilibrium is reached. The rotation of C will induce a moment at B.

Repeat this process until moment equilibrium is reached at the nodes.

Assume that the sum of the initial moments at the node B is equal to M_0.

Rotation will take place until moment equilibrium is attained, i.e., sum moments $\Sigma M_B = 0$.



Therefore: $M^D_{BA} + M^D_{BC} + M_0 = 0$

Where M^D_{BA} *and* M^D_{BC} are the moments as a result of the rotation at B, θ_B, and are called the distribution moments. Remember that all the other rotations and sway are prevented.

$M^D_{BA} = \frac{2 \cdot EI_{AB}}{L_{AB}} \cdot (2 \cdot \theta_B) = \frac{4 \cdot EI_{AB} \cdot \theta_B}{L_{AB}}$. In a similar fashion $M^D_{BC} = \frac{2 \cdot EI_{BC}}{L_{BC}} \cdot (2 \cdot \theta_B) = \frac{4 \cdot EI_{BC} \cdot \theta_B}{L_{BC}}$

But: $M^D_{BA} + M^D_{BC} + M_0 = 0$

Solve for θ_B.

$$\theta_B = -\frac{M_0}{\frac{4 \cdot EI_{AB}}{L_{AB}} + \frac{4 \cdot EI_{BC}}{L_{BC}}}$$

Solve the distribution moments.

$$M_{BA}^{D}=-\frac{\frac{4\cdot EI_{AB}}{L_{AB}}\cdot M_0}{\frac{4\cdot EI_{AB}}{L_{AB}}+\frac{4\cdot EI_{BC}}{L_{BC}}}=-\frac{k_{BA}\cdot M_0}{k_{BA}+k_{BC}}=-\frac{k_{BA}\cdot M_0}{\sum k_B}$$

$$M_{BC}^{D}=-\frac{\frac{4\cdot EI_{BC}}{L_{BC}}\cdot M_0}{\frac{4\cdot EI_{AB}}{L_{AB}}+\frac{4\cdot EI_{BC}}{L_{BC}}}=-\frac{k_{BC}\cdot M_0}{k_{BA}+k_{BC}}=-\frac{k_{BC}\cdot M_0}{\sum k_B}$$

k_{BA} is the stiffness of the member BA at the node B. It is also the moment that would be induced if a unit rotation were applied at B in the member BA and the rotation at A was zero.

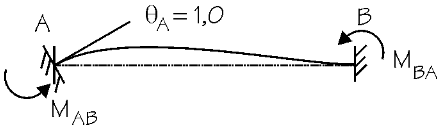
If B rotates a bending moment will be induced at A and C. Assume a rotation θ_B and calculate the moment at A.

$M_{AB}^{D}=\frac{2\cdot EL_{AB}}{L_{AB}}\cdot\left(\theta_B\right)$, but bear in mind that $\theta_B=-\frac{M_0}{\frac{4\cdot EI_{AB}}{L_{AB}}+\frac{4\cdot EI_{BC}}{L_{BC}}}$

$$M_{AB}^{D}=-\frac{\frac{2\cdot EI_{AB}}{L_{AB}}\cdot M_0}{\frac{4\cdot EI_{AB}}{L_{AB}}+\frac{4\cdot EI_{BC}}{L_{BC}}}$$

The distributed bending moment is half the value of the distributed bending moment at B. This is called the carry-over factor, C_{BA} = ½.

The same solution may be obtained if one remembers that the stiffness of a member is the moment that is induced if a unit rotation is applied at the node.



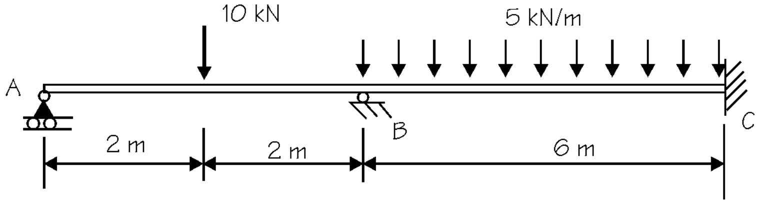
$$M_{AB}=k_{AB}=\frac{2\cdot EI_{AB}}{L_{AB}}\cdot\left(2\cdot\theta_A\right)=\frac{2\cdot EI_{AB}}{L_{AB}}\cdot\left(2\cdot 1{,}0\right)=\frac{4\cdot EI_{AB}}{L_{AB}}$$

$$M_{BA}=\frac{2\cdot EI_{AB}}{L_{AB}}\cdot\left(\theta_A\right)=\frac{2\cdot EI_{AB}}{L_{AB}}\cdot\left(1{,}0\right)=\frac{2\cdot EI_{AB}}{L_{AB}}$$

$$C_{AB}=\frac{M_{BA}}{M_{AB}}=\frac{1}{2}$$

Example:

Use the method of moment-distribution to determine the bending moment diagramme of the following beam.



Distribution at A and B

Stiffness of members at A:

$$k_{AB} = \frac{4 \cdot EI_{AB}}{L_{AB}} = \frac{4 \cdot EI}{4} = 1{,}0$$

$$\sum k = 1{,}0$$

Distribution Factors

$$D_{AB} = \frac{k_{AB}}{\sum k} = \frac{1{,}0}{1{,}0} = 1{,}0$$

Stiffness of members at B:

$$k_{BA} = \frac{4 \cdot EI_{BA}}{L_{AB}} = \frac{4 \cdot EI}{4} = 1{,}0$$

$$k_{BC} = \frac{4 \cdot EI_{BC}}{L_{BC}} = \frac{4 \cdot EI}{6} = 0{,}66667$$

$$\sum k = 1{,}66667$$

$$D_{AB} = \frac{k_{AB}}{\sum k} = \frac{1{,}0}{1{,}66667} = 0{,}60$$

$$D_{BC} = \frac{k_{BC}}{\sum k} = \frac{0{,}66667}{1{,}66667} = 0{,}40$$

Initial Moments:

$$M^0_{AB} = FEM_{AB} = \frac{W \cdot L}{8} = \frac{10 \cdot 4}{8} = +5{,}0\ kN.m$$

$$M^0_{BA} = FEM_{BA} = -\frac{W \cdot L}{8} = -\frac{10 \cdot 4}{8} = -5{,}0\ kN.m$$

$$M^0_{BC} = FEM_{BC} = +\frac{w \cdot L^2}{12} = +\frac{5 \cdot 6^2}{12} = 15{,}0\ kN.m$$

$$M^0_{CB} = FEM_{CB} = -\frac{w \cdot L^2}{12} = -\frac{5 \cdot 6^2}{12} = -15{,}0\ kN.m$$

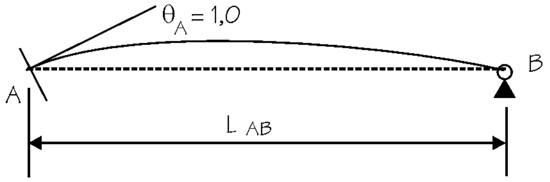
Distribution of the moments:

Carry over factors ½ ½

Action	M_{AB}	M_{BA}	M_{BC}	M_{CB}
Distribution factors	1,00	0,60	0,40	
Initial moments	+ 5,000	- 5,000	+ 15,000	- 15,000
Allow rotation of A	- 5,000 →	½ - 2,500		
Allow rotation of B	- 2,250 ½ ←	- 4,500	-3,000 →	½ -1,500
Allow rotation of A	+ 2,250	+ 1,125		
Allow rotation of B	- 0,3375	- 0,675	- 0,450	- 0,225
Allow rotation of A	+ 0,3375	+ 0,1688		
Allow rotation of B	- 0,051	- 0,1013	- 0,0675	- 0,0338
Allow rotation of A	+ 0,051	+ 0,0255		
		- 0,0153	- 0,0102	
	0,000	- 11,472	+ 11,472	- 16,759

Members with a hinge on one side:

Assume a member with a hinge at B.



Stiffness = moment required to induce a unit rotation at A:

$$k_{AB} = M_{AB} = \frac{3 \cdot EI_{AB}}{L_{AB}}(\theta_A) = \frac{3 \cdot EI_{AB}}{L_{AB}}$$

$$M_{AB}^0 = FEM_{AB} - \frac{1}{2} FEM_{BA}$$

Redo example 1 using the stiffness of a member with a hinge.

$$k_{BA} = \frac{3 \cdot EI_{BA}}{L_{AB}} = \frac{3 \cdot EI}{4} = 0{,}750 \qquad D_{AB} = \frac{k_{AB}}{\sum k} = \frac{0{,}750}{1{,}41667} = 0{,}52941$$

$$k_{BC} = \frac{4 \cdot EI_{BC}}{L_{BC}} = \frac{4 \cdot EI}{6} = 0{,}66667 \qquad D_{BC} = \frac{k_{BC}}{\sum k} = \frac{0{,}66667}{1{,}41667} = 0{,}47059$$

$$\sum k = 1{,}41667$$

$$M_{BA}^0 = FEM_{BA} - \frac{1}{2} FEM_{AB} = -\frac{W \cdot L}{8} - \frac{1}{2} \cdot \left(+ \frac{W \cdot L}{8} \right) = -7{,}50\ kN.m$$

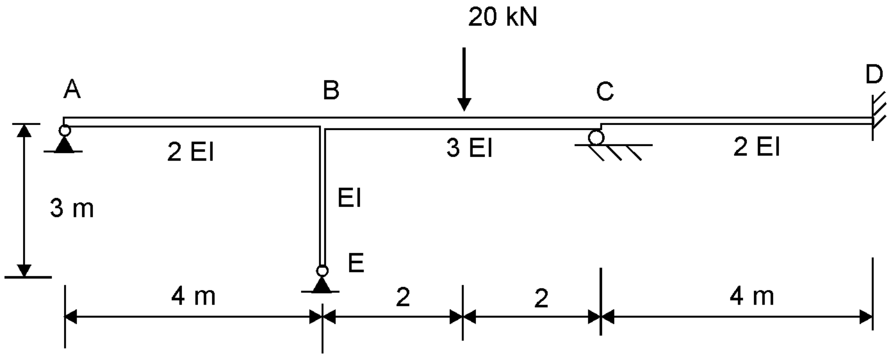
$$M_{BC}^0 = FEM_{BC} = +\frac{w \cdot L^2}{12} = +\frac{5 \cdot 6^2}{12} = 15{,}0\ kN.m$$

$$M_{CB}^{0} = FEM_{CB} = -\frac{w\cdot L^2}{12} = -\frac{5\cdot 6^2}{12} = -15{,}0\ kN.m$$

	M_{BA}	M_{BC}	M_{CB}
Dist Factors	0,52941	0,47059	
Init moments	- 7,500	+ 15,000	- 15,000
Rotate B	- 3,9706	- 3,5294	- 1,7647
	-11,4706	+ 11,4706	-16,7647

Example 3:

Determine the bending moment diagramme of the following structure.



Rotation will occur at B and C.

Stiffness at B

$$k_{BA} = \frac{3\cdot EI_{BA}}{L_{AB}} = \frac{3\cdot 2EI}{4} = 1{,}5 \qquad D_{BA} = \frac{k_{BA}}{\sum k} = \frac{1{,}5}{5{,}5} = 0{,}2727$$

$$k_{BE} = \frac{3\cdot EI_{BE}}{L_{BE}} = \frac{3\cdot EI}{3} = 1{,}0 \qquad D_{BE} = \frac{k_{BE}}{\sum k} = \frac{1{,}0}{5{,}5} = 0{,}1818$$

$$k_{BC} = \frac{4\cdot EI_{BC}}{L_{BC}} = \frac{4\cdot 3EI}{4} = 3{,}00 \qquad D_{BC} = \frac{k_{BC}}{\sum k} = \frac{3{,}0}{5{,}5} = 0{,}5455$$

$$\sum k = 5{,}5$$

$$k_{CB} = \frac{4\cdot EI_{CB}}{L_{CB}} = \frac{4\cdot 3EI}{4} = 3{,}0 \qquad D_{CB} = \frac{k_{CB}}{\sum k} = \frac{3{,}0}{5{,}0} = 0{,}600$$

$$k_{CD} = \frac{4\cdot EI_{CD}}{L_{CD}} = \frac{4\cdot 2EI}{4} = 2{,}0 \qquad D_{CD} = \frac{k_{CD}}{\sum k} = \frac{2{,}0}{5{,}0} = 0{,}400$$

$$\sum k = 5{,}0$$

Initial moments:

$$M_{BC}^{0} = FEM_{BC} = +\frac{W\cdot L}{8} = +\frac{20\cdot 4}{8} = +10{,}0\ kN.m$$

$$M_{CB}^{0} = FEM_{CB} = -\frac{W \cdot L}{8} = -\frac{10 \cdot 4}{8} = -10{,}0\ kN.m$$

M_{AB}	M_{BE}	M_{BC}	M_{CB}	M_{CD}	M_{DC}
0,2727	0,1818	0,5455	0,600	0,400	
		+10,0000	-10,000		
-2,727	-1.818	-5.455	-2,728		
		+3,818	+7,637	+5,091	+2,545
-1,041	-0,694	-2,083	-1,041		
		+0,312	+0,625	+0,416	+0,208
-0,085	-0,057	-0,170	-0,085		
			+0,051	+0,034	
-3,853	-2,569	+6,422	-5,541	+5,541	+2,753

Structural Frames with Sway.

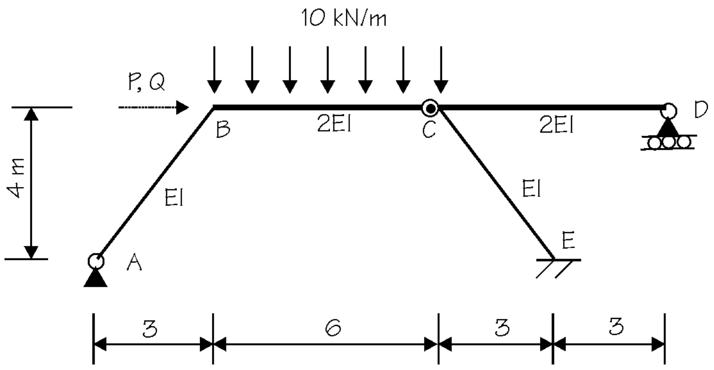
Frames with a sway mechanism may be tackled by preventing the sway and calculating the force required to prevent the sway, call this P. Arbitrary sway is then applied to the structure and the force that leads to the arbitrary sway is calculated, call this Q. Apply the super-position equation as neither of the forces are really there.

P + x Q = 0

Final bending moment = Bending moment with sway prevented + x times bending moment with arbitrary sway.

Example 4:

Determine the bending moment diagramme of the following sway structure. The support A is a hinge, E is fixed and D is a roller. There is a hinge in BC at C.



Force P prevents the sway and force Q induces the arbitrary sway.
Apply force P to prevent the sway. Rotation will occur at B and at C.

At B

$$k_{BA} = \frac{3 \cdot EI_{AB}}{L_{AB}} = \frac{3 \cdot EI}{5} = 0{,}6 \qquad D_{BA} = \frac{k_{BA}}{\sum k} = \frac{0{,}6}{1{,}6} = 0{,}375$$

$$k_{BC} = \frac{3 \cdot EI_{BC}}{L_{BC}} = \frac{3 \cdot 2EI}{6} = 1{,}0 \qquad D_{BC} = \frac{k_{BC}}{\sum k} = \frac{1{,}0}{1{,}6} = 0{,}625$$

$$\sum k = 1{,}6 \qquad \sum D = 1{,}000$$

At C

$$k_{CD} = \frac{3 \cdot EI_{CD}}{L_{CD}} = \frac{3 \cdot 2EI}{6} = 1{,}00 \qquad D_{CD} = \frac{k_{CD}}{\sum k} = \frac{1{,}0}{1{,}8} = 0{,}5556$$

$$k_{CE} = \frac{4 \cdot EI_{CE}}{L_{CE}} = \frac{4 \cdot EI}{5} = 0{,}8 \qquad D_{CE} = \frac{k_{CE}}{\sum k} = \frac{0{,}8}{1{,}8} = 0{,}4444$$

$$\sum k = 1{,}8 \qquad \sum D = 1{,}000$$

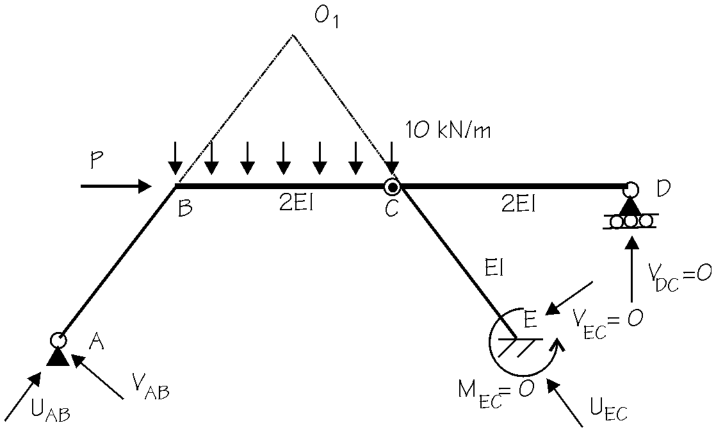
Initial moments:

$$M_{BC}^{0} = FEM_{BC} - \tfrac{1}{2} FEM_{CB}$$

$$M_{BC}^{0} = \frac{w \cdot L^2}{12} - \tfrac{1}{2}\left(\frac{-w \cdot L^2}{12}\right) \quad = \quad \frac{10 \cdot 6^2}{12} + \frac{10 \cdot 6^2}{2 \cdot 12} = +45{,}00\ kN.m$$

M_{BA}	M_{BC}	M_{CD}	M_{CE}	M_{EC}
0,375	0,625	0,5556	0,4444	
	+45,000			
-16,875	-28,125			
-16,875	+16,875	0	0	0

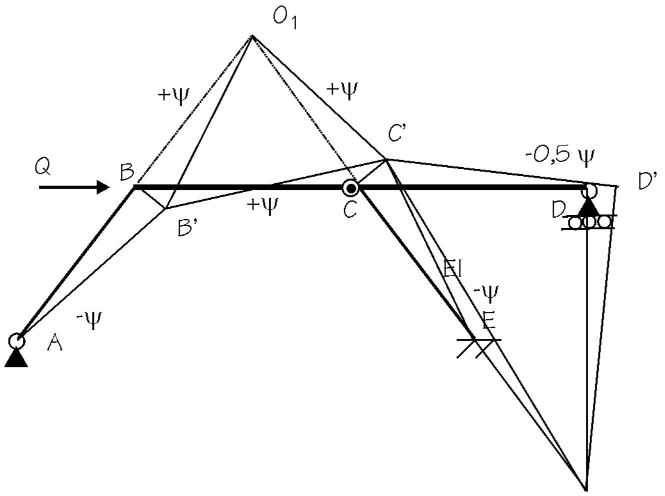
Force, P, that prevents sway:



Take moments about O_1.
$\Sigma M_{O1} = 0$. $\quad - Px4 + V_{AB} \times 10 = 0$

$$V_{AB} = \frac{-M_{BA}}{L_{AB}} = \frac{-16{,}875}{5} = -8{,}4375\ kN$$

Arbitrary Sway



Choose ψ_{AB} as the unknown angle and calculate all others in terms of this angle.

$$BB' = 5 \cdot \psi \qquad \psi_{O_1B} = \psi_{BC} = \frac{BB'}{5} = \psi \qquad CC' = 5 \cdot \psi \qquad \psi_{O_2C} = \psi_{CD} = \frac{CC'}{10} = -0{,}5 \cdot \psi$$

Assuming that all rotation angles are equal to 0 and there are sway angle it is possible to write the initial moments in terms of these sway angle.

Standard case: $M_{AB} = \frac{2 \cdot EI_{AB}}{L_{AB}}(2 \cdot \theta_A + \theta_B - 3 \cdot \psi_{AB})$ with θ angles = 0

$$M_{AB} = \frac{-6 \cdot EI_{AB}(\psi_{AB})}{L_{AB}}$$

Modified form: $M_{AB} = \frac{3 \cdot EI_{AB}}{L_{AB}}(\theta_A - \psi_{AB})$ with θ angles = 0

$$M_{AB} = \frac{-3 \cdot EI_{AB}(\psi_{AB})}{L_{AB}}$$

Therefore:

$M_{BA} : M_{BC} : M_{CD} : M_{CE} : M_{EC}$

$$-\frac{3 \cdot EI_{AB} \cdot \psi_{AB}}{L_{AB}} : -\frac{3 \cdot EI_{BC} \cdot \psi_{BC}}{L_{BC}} : -\frac{3 \cdot EI_{CD} \cdot \psi CD}{L_{CD}} : -\frac{6 \cdot EI_{CE} \cdot \psi_{CE}}{L_{CE}} : -\frac{6 \cdot EI_{CE} \cdot \psi_{CE}}{L_{CE}}$$

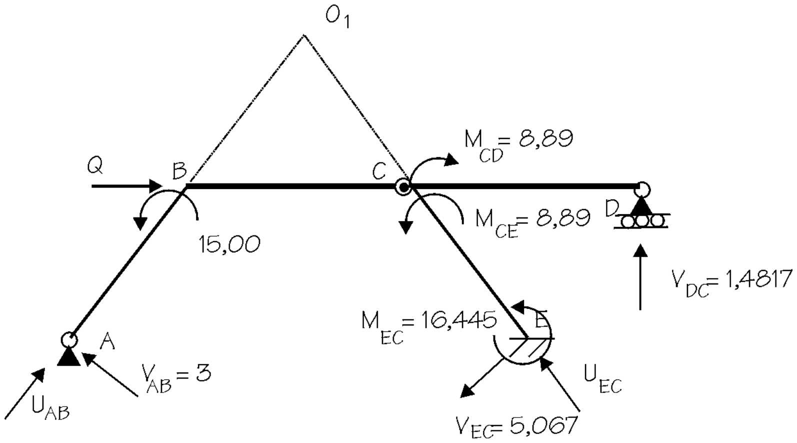
$$-\frac{3 \cdot EI \cdot (-\psi)}{5} : -\frac{3 \cdot 2EI \cdot (+\psi)}{6} : -\frac{3 \cdot 2EI \cdot (-0{,}5 \cdot \psi)}{6} : -\frac{6 \cdot EI \cdot (-\psi)}{5} : -\frac{6 \cdot EI \cdot (-\psi)}{5}$$

0,6 EI : -1,0 EI : 0,5 EI : 1,2 EI : 1,2 EI

Set EI = 20

12 : -20 : 10 : 24 : 24

M_{BA}	M_{BC}	M_{CD}	M_{CE}	M_{EC}
0,375	0,625	0,5556	0,4444	
12,000	-20,000	+10,000	+24,000	+24,00
+3,000	+5,000	-18,890	-15,110	-7,555
+15,000	-15,000	-8,890	+8,890	+16,445



$\sum M_{O1} = 0$

$- 4 \times Q + 3 \times 10 – 16,445 + 5,067 \times 10 – 1,4817 \times 9 = 0$

$Q = 12,7237$ kN

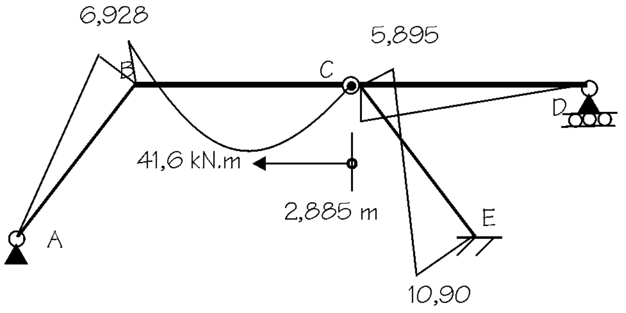
Superposition equation:

$P + x Q = 0$
$X = 0,66313$

Final bending moments:

$M_F = M_{\text{sway prevented}} + x M_{\text{arbitrary sway}}$

M_{BA}	M_{BC}	M_{CD}	M_{CE}	M_{EC}	
-16,875	+16,875	0	0	0	$M_{\text{sway prevented}}$
+9,947	-9,947	-5,895	+5,895	+10,905	$x M_{\text{arbitrary sway}}$
-6,928	+6,928	-5,895	+5,895	+10,905	M_F

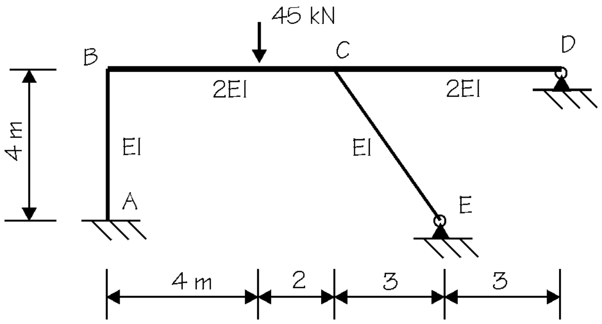


Final Bending Moment Diagramme.

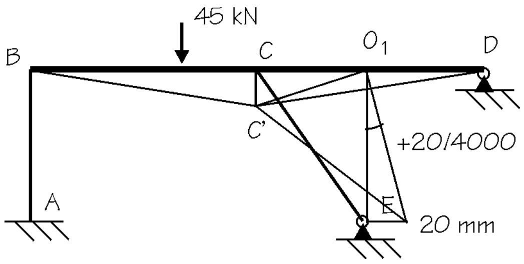
Structure with Displacement of a Support

Example 5:

Determine the bending moment diagramme of the structure if E = 200 GPa, I = 150 x 10^{-6} m^4 and the support E moves 20 mm to the right.



View the structure with the displacement of the support.



EE' = 20 mm, therefore: $\psi_{O1B} = \frac{+20}{4000} = \psi_{O1C} = \psi_{CE}$

$$CC' = \psi_{O1C} \cdot 3000 = \frac{20}{4000} \cdot 3000 = 15\ mm$$

$$\psi_{BC} = \frac{CC'}{6000} = \frac{-15}{6000} \qquad \psi_{DC} = \frac{CC'}{6000} = \frac{15}{6000}$$

Use the slope-deflection equations to determine the initial moments with all rotations θ equal to zero.

$$M_{BC}^{0} = \frac{2 \cdot EI_{BC}}{L_{BC}} \left(2 \cdot \theta_B + \theta_C - 3 \cdot \psi_{BC} \right) + FEM_{BC}$$

$$M_{BC}^{0} = \frac{2 \cdot 2EI}{6} \left(-3 \cdot \psi_{BC} \right) + FEM_{BC} = \frac{2 \cdot 2 \cdot 200x10^{6} \cdot 150x10^{-6}}{6} \left(-3 \cdot \frac{-15}{6000} \right) + \frac{45 \cdot 4 \cdot 2^{2}}{6^{2}}$$

$$M_{BC}^{0} = 150 + 20 = 170 \; kN.m$$

$$M_{CB}^{0} = \frac{2 \cdot 2EI}{6} \left(-3 \cdot \psi_{BC} \right) + FEM_{CB} = \frac{2 \cdot 2 \cdot 200x10^{6} \cdot 150x10^{-6}}{6} \left(-3 \cdot \frac{-15}{6000} \right) + \frac{-45 \cdot 4^{2} \cdot 2}{6^{2}}$$

$$M_{CB}^{0} = 150 - 40 = 110 \; kN.m$$

$$M_{CD}^{0} = \frac{3 \cdot 2EI}{6} \left(-\psi_{CD} \right) = \frac{3 \cdot 2 \cdot 200x10^{6} \cdot 150x10^{-6}}{6} \left(-\frac{15}{6000} \right)$$

$$M_{CD}^{0} = -75 \; kN.m$$

$$M_{CE}^{0} = \frac{3 \cdot EI}{5} \left(-\psi_{CE} \right) = \frac{3 \cdot 200x10^{6} \cdot 150x10^{-6}}{5} \left(-\frac{20}{4000} \right)$$

$$M_{CE}^{0} = -90 \; kN.m$$

At B

$$k_{BA} = \frac{4 \cdot EI_{AB}}{L_{AB}} = \frac{4 \cdot EI}{4} = 1{,}0 \qquad D_{BA} = \frac{k_{BA}}{\sum k} = \frac{1{,}0}{2{,}3333} = 0{,}4286$$

$$k_{BC} = \frac{4 \cdot EI_{BC}}{L_{BC}} = \frac{4 \cdot 2EI}{6} = 1{,}3333 \qquad D_{BC} = \frac{k_{BC}}{\sum k} = \frac{1{,}3333}{2{,}3333} = 0{,}5714$$

$$\sum k = 2{,}3333 \qquad \sum D = 1{,}000$$

At C

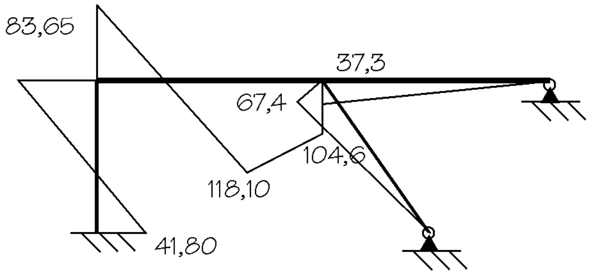
$$k_{CB} = \frac{4 \cdot EI_{CB}}{L_{CB}} = \frac{4 \cdot 2EI}{6} = 1{,}3333 \qquad D_{CB} = \frac{k_{CB}}{\sum k} = \frac{1{,}3333}{2{,}9333} = 0{,}4546$$

$$k_{CD} = \frac{3 \cdot EI_{CD}}{L_{CD}} = \frac{3 \cdot 2EI}{6} = 1{,}000 \qquad D_{CD} = \frac{k_{CD}}{\sum k} = \frac{1{,}000}{2{,}9333} = 0{,}3409$$

$$k_{CE} = \frac{3 \cdot EI_{CE}}{L_{BCE}} = \frac{3 \cdot EI}{5} = 0{,}6000 \qquad D_{CE} = \frac{k_{CE}}{\sum k} = \frac{0{,}6}{2{,}9333} = 0{,}2045$$

$$\sum k = 2{,}9333 \qquad \sum D = 1{,}000$$

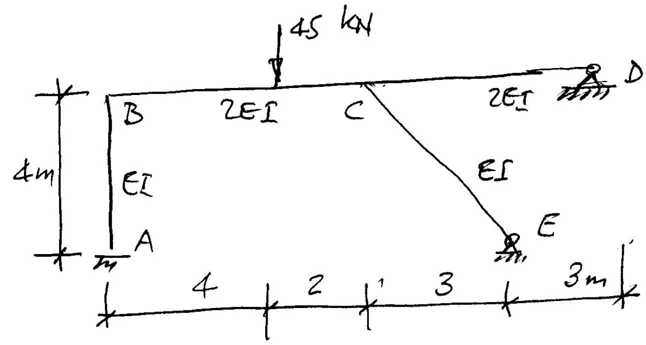
M_{AB}	M_{BA}	M_{BC}	M_{CB}	M_{CD}	M_{CE}
	0,4286	0,5714	0,4546	0,3409	0,2045
		+170,000	+110,000	-75,000	-90,000
-36,431	-72,862	-97,138	-48,569		
		+23,541	+47,082	+35,307	+21,180
-5,045	-10,090	-13,451	-6,726		
		+1,529	+3,058	+2,293	+1,375
-0,328	-0,655	-0,874	-0,437		
		+0,099	+0,199	+0,149	+0,089
	-0,042	-0,057			
-41,804	-83,649	+83,649	+104,607	-37,251	-67,356

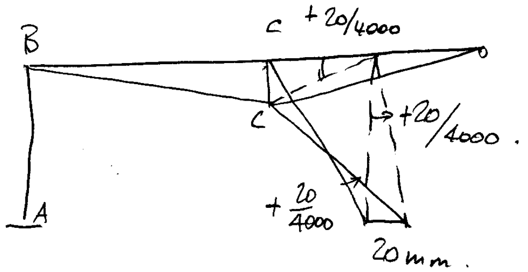


Final Bending Moment Diagramme

EXAMPLE: DISPLACEMENT OF SUPPORT

Determine the bending momentdiagramme of the structure if $E = 200$ GPa, $I = 150 \times 10^{-6}$ m^4 and the support E moves 20 mm to the right





$$CC' = \frac{20}{4000} * 3000$$
$$= 15 \text{mm}$$
$$\psi_{BC} = \frac{-15}{6000}$$
$$\psi_{DC} = \frac{+15}{6000}$$

$$M^0_{BC} = \frac{2EI_{BC}}{L_{BC}}\left(-3\psi_{BC}\right) + FEM_{BC} = \frac{2 \times 2 \times 200 \times 10^{+6} * 150 \times 10^{6}}{6}\left(-3 * \frac{-15}{6000}\right) + \frac{45 * 4 \times 2^2}{6^2}$$
$$= 150 + 20 = 170 \text{ kN.m}$$

$$M^0_{CB} = \frac{2EI_{BC}}{L_{BC}}\left(-3\psi_{BC}\right) + FEM_{CB} = \frac{2 \times 2 \times 200 \times 10^6 \times 150 * 10^{-6}}{6}\left(-3 \times \frac{-15}{6000}\right) - \frac{45 \times 4^2 \times 2}{6^2}$$
$$= 150 - 40 = 110 \text{ kN.m}$$

$$M_{CD} = \frac{3.EI_{CD}}{L_{CD}}\left(-\psi_{CD}\right) = \frac{3 \times 2 \times 200 \times 10^6 * 150 \times 10^{-6}}{6}\left(-\frac{15}{6000}\right) = -75$$

$$M_{CE} = \frac{3.EI_{CE}}{L_{CE}}\left(-\psi_{CE}\right) = \frac{3 \times 200 \times 10^6 \times 150 * 10^{-6}}{5}\left(\frac{-20}{4000}\right) = -90$$

@ B

$$k_{BA} = \frac{4.EI_{BA}}{L_{BA}} = \frac{4.EI}{4} = 1.0 \qquad D_{BA} = \frac{k_{BA}}{\Sigma k} = \frac{1}{2,3333} = 0,4286$$

$$k_{BC} = \frac{4.EI_{BC}}{L_{BC}} = \frac{4 \times 2.EI}{6} = \frac{1,3333}{2,3333} \qquad D_{BC} = \frac{k_{BC}}{\Sigma k} = \frac{1,3333}{2,3333} = \frac{0,5714}{1,0000}.$$

@ C

$$k_{CB} = \frac{4.EI_{CB}}{L_{CB}} = \frac{4.2}{6} = 1,3333. \qquad D_{CB} = \frac{1,3333}{2,9333} = 0,4546$$

$$k_{CD} = \frac{3.EI_{CD}}{L_{CD}} = \frac{3.2}{6} = 1,0000 \qquad D_{CD} = \frac{1,000}{2,9333} = 0,3409$$

$$k_{CE} = \frac{3.EI_{CE}}{L_{CE}} = \frac{3.1}{5} = \frac{0,6000}{2,9333} \qquad D_{CE} = \frac{0,6000}{2,9333} = \frac{0,2045}{1,0000}$$

$\frac{1}{2}$ $\frac{1}{2}$

M_{AB}	M_{BA}	M_{BC}	M_{CB}	M_{CD}	M_{CE}
	0,4286	0,5714	0,4546	0,3409	0,2045
		+170,000	+110,000	-75,000	-90,000
-36,431	-72,862	-97,138	-48,569		
		+23,541	+47,082	+35,307	+21,180
-5,045	-10,090	-13,451	-6,726		
		+1,529	+3,058	+2,293	+1,375
-0,328	-0,655	-0,874	-0,437		
		+0,099	+0,199	+0,149	+0,089
	-0,042	-0,057			
-41,804	-83,649	+83,649	104,607	-37,251	-67,356